Scalar (Dot Product)

$\vec{a} \cdot \vec{b}$ = Scalar

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$

Scalar

$0 \leq \theta \leq \pi$

Important → Rules

(i) $\theta = 0^\circ$

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|$$

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos 0^\circ$$

(ii) $\theta = 90^\circ$

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos 90^\circ$$

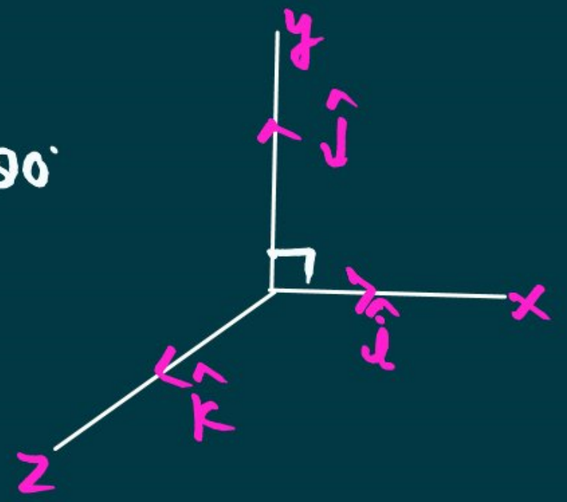
$$\vec{a}\cdot\vec{b} = 0$$

(iii) $\theta = \pi = 180^\circ$

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos 180^\circ$$

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|(-1)$$

$$\vec{a}\cdot\vec{b} = -|\vec{a}||\vec{b}|$$



(iv)

$$\vec{a}\cdot\vec{a} = |\vec{a}||\vec{a}|\cos 0^\circ = |\vec{a}||\vec{a}| = |\vec{a}|^2 = a^2$$

Square and Scalar product of $\hat{i}, \hat{j}, \hat{k}$

$\theta = 0$

$$\hat{i}\cdot\hat{i} = 1$$
$$\hat{j}\cdot\hat{j} = 1$$
$$\hat{k}\cdot\hat{k} = 1$$

$\theta = 0$

$\theta = 90^\circ$

$$\hat{i}\cdot\hat{j} = 0$$
$$\hat{i}\cdot\hat{k} = 0$$
$$\hat{j}\cdot\hat{k} = 0$$

Find the Angle between two Vectors $\vec{a}, \vec{b}$

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$$

Question 1.

(i) If $\vec{a}$ is a unit vector and $(\vec{x}-\vec{a})\cdot(\vec{x}+\vec{a}) = 15$, then find $|\vec{x}|$.

(ii) If $\vec{p}$ is a unit vector and $(\vec{x}-\vec{p})\cdot(\vec{x}+\vec{p}) = 80$, then find $|\vec{x}|$.

$|\vec{a}| = 1$

$(\vec{x}-\vec{a})\cdot(\vec{x}+\vec{a}) = 15$

$|\vec{x}|^2 - |\vec{a}|^2 = 15$

$|\vec{x}|^2 - 1 = 15$

$|\vec{x}|^2 = 15+1$

$|\vec{x}|^2 = 16$

$|\vec{x}| = \sqrt{16}$

$|\vec{x}| = \pm 4$

(ii) $|\vec{p}|=1$

$|\vec{x}-\vec{p}|\cdot|\vec{x}+\vec{p}|=80$ Find $|\vec{x}|$

$|\vec{x}|^2-|\vec{p}|^2=80$

$|\vec{x}|^2-1=80$

$|\vec{x}|^2=80+1$

$|\vec{x}|^2=81$

$|\vec{x}|=\sqrt{81}$

$\boxed{|\vec{x}|=9}$

Question 2.

(i) If $(\vec{a}-\vec{b})\cdot(\vec{a}+\vec{b}) = 12$ and $|\vec{a}| = 2|\vec{b}|$, then find $|\vec{a}|$ and $|\vec{b}|$.

(ii) If $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $\vec{a}\cdot\vec{b} = 1$, then find $(\vec{a}-\vec{b})^2$

(iii) If $|\vec{a}| = 2$, $|\vec{b}| = 5$ and $\vec{a}\cdot\vec{b} = 8$, then find $|\vec{a}-\vec{b}|$.

(iv) If $|\vec{a}| = 3$, $|\vec{b}| = 5$ and $\vec{a}\cdot\vec{b} = -8$ then find $|\vec{a}+\vec{b}|$.

$(\vec{a}-\vec{b})\cdot(\vec{a}+\vec{b}) = 12$ $\quad |\vec{a}| = 2|\vec{b}|$ $\quad$ $|\vec{a}|$ and $|\vec{b}|$

$|\vec{a}|^2 - |\vec{b}|^2 = 12$

$4|\vec{b}|^2 - |\vec{b}|^2 = 12$

$3|\vec{b}|^2 = 12$

$|\vec{b}|^2 = \frac{12}{3}$

$|\vec{b}|^2 = 4$

$|\vec{b}| = \sqrt{4}$

$\boxed{|\vec{b}| = \pm 2}$

$\boxed{|\vec{a}| = 2 \times 2 = 4}$

(ii) $|\vec{a}|=3 \quad |\vec{b}|=4$ and $\vec{a}\cdot\vec{b}=1$ $(\vec{a}-\vec{b})^2$

$$(\vec{a}-\vec{b})^2 = |\vec{a}|^2+|\vec{b}|^2-2|\vec{a}||\vec{b}|$$

$$= 9+16-2\times 1$$

$$\boxed{= 25-2=23}$$

(iii) $|\vec{a}|=2 \quad |\vec{b}|=5 \quad \vec{a}\cdot\vec{b}=8 \quad |\vec{a}-\vec{b}|$

$$|\vec{a}-\vec{b}|^2 = |\vec{a}|^2+|\vec{b}|^2-2|\vec{a}||\vec{b}|$$

$$= (2)^2+(5)^2-2\times 8$$

$$= 4+25-16$$

$$= 29-16$$

$$|\vec{a}-\vec{b}|^2 = 13$$

$$\boxed{|\vec{a}-\vec{b}| = \sqrt{13}}$$

$|\vec{a}| = 3 \quad |\vec{b}| = 5 \quad \vec{a} \cdot \vec{b} = -8 \quad |\vec{a} + \vec{b}|$

$$|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|$$

$$= (3)^2 + (5)^2 + 2 \times -8$$

$$= 9 + 25 - 16$$

$$= 34 - 16$$

$$|\vec{a}+\vec{b}|^2 = 20$$

$$\boxed{|\vec{a}+\vec{b}| = \sqrt{20}}$$

$=$